

THE HARDEST MATH PROBLEM STUDENT CONTEST

CHALLENGE 2 ANSWER KEY

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GRADE 6

Step 1

Convert Sonia's training swim time into seconds:
13 min 45 sec = 825 sec for 750 m

Step 2

Set up two equivalent proportions comparing the old and new swim length distances and times:
 $750 \text{ m} / 825 \text{ sec} = 425 \text{ m} / x \text{ sec}$

Step 3

Cross multiply or solve algebraically to find that
 $x = 467.5$ seconds.

Step 4

Convert 467.5 seconds to minutes:
= 7 minutes 47.5 seconds is the new swim time

Step 5

Add the new swim time time plus the training times for the bike and run: 7 min 47.5 sec + 45 min + 19 min = 71 min and 47.5 sec; or 4,307.5 sec total

Step 6

Sonia would need to complete the race 1 min 47.5 seconds faster, or 107.5 seconds faster. So the run must be completed in 17 min 12.5 seconds.

Step 7

Set up the proportion to compare the training run time to the new run time needed.
 $17 \text{ min } 12.5 \text{ sec} / 19 \text{ min} = x \% / 100\%$
Or $1032.5 \text{ secs} / 1140 \text{ secs} = x \% / 100\%$

$x = 90.57\%$. Rounding to the nearest tenth of a percent yields 90.6%.

Sonia would need to finish her run in 90.6% of the time of her training run.

Grade 7

Step 1

First, determine how much of a lead Sonia has after the swim.
Sonia's training time was 13 min 45 sec = 13.75 min, or 825 seconds for 750 m.

Find Sonia's new swim length time:

$$750 \text{ m} / 825 \text{ secs} = 425 \text{ m} / x \text{ secs}$$

$$x = 467.5 \text{ seconds}$$

Convert 467.5 seconds to minutes: $467.5 / 60 = 7.79167$ minutes.

That's 7 minutes and .79167 of a minute.

$$.79167 * 60 \text{ seconds} = 47.496 \text{ seconds}$$

= 7 minutes 47.5 seconds for Sonia's new swim length time

Stephen's training swim time was 16 min 15 sec for 750 m.

$$750 \text{ m} / 975 \text{ secs} = 425 \text{ m} / x \text{ secs}$$

$$x = 552.5 \text{ seconds, which is } \mathbf{9 \text{ min } 12.5 \text{ secs}}$$

After the swim, Stephen's time is at 7 min 23 sec + 9 min 12.5 sec = 16 min 35.5 sec. Subtracting Sonia's time of 7 min 47.5 sec from Stephen's time, **Sonia has a lead of 8 min 48 sec after the swim.**

Step 2

Recognize that Stephen will not catch up during bike race...

For Sonia: 20 km took 45 min

For Stephen: 20 km took 40 min

Stephen's biking was 5 minutes faster than Sonia's. Her lead of 8 min and 48 seconds, minus those 5 minutes, means Sonia's lead is now 3 min 48 sec (or 3.8 min lead).

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(Grade 7 continued)**Step 3**

Finding unit rates:

Sonia's running rate is 3.8 min per km (derived from 19 min/5 km), and Stephen's rate is 3 min per km (derived from 15 min/5 km).

With Sonia's 3.8 min lead, set up a proportion showing when her time will equalize with Stephen's:

$$\begin{array}{r} 3.8x \text{ (min/km)} = 3.0x \text{ (min/km)} + 3.8 \text{ min} \\ -3.0x \qquad \qquad -3.0x \\ \hline 0.8x \text{ (min/km)} = \qquad \qquad \qquad 3.8 \text{ min} \\ x = 4.75 \text{ km} \end{array}$$

Stephen will catch up after running for 4.75 km.

Step 4

Recognize the total distance traveled over the course of the triathlon is actually:

$$425 \text{ m (which is .425 km)} + 20 \text{ km} + 4.75 \text{ km} = 25.175 \text{ km}$$

Step 5

Recognize the final answer must be in meters.

25.175 km converted to meters is 25,175 meters.

It will take 25,175 meters for Stephen to catch up to Sonia during the triathlon.

Grade 8**Step 1**

The swimmers outnumbered the ducklings by five times in the lake.

Let w = swimmers

Let d = ducklings

$$w = 5d$$

Step 2

When Sonia finished swimming, she was the 126th person to leave the water. There were now twice as many ducklings as swimmers, which can be represented as:

$$2(w - 126) = d$$

Step 3

Recognize therefore that $2(5d - 126) = d$

$$10d - 252 = d$$

$$9d = 252$$

$$d = 28 \text{ ducklings}$$

If 5 times as many ducklings as swimmers were in the lake, then **there were 140 swimmers in the lake when the last competitor jumped in.**