THE HARDEST MATH PROBLEM
CHALLENGE 2 ANSWER KEY — GRADE 6

Although the problem has one correct numeric solution, there are multiple pathways students can take to arrive at the answer.

Step 1: I begin by translating some of the most important words into an equation.

"...the ***average*** number of colony collapses due to pesticides from 2007–2012 ***compared*** to the average from 2002–2007 **was 1 ²/₃ times** the **ratio of the averages** of incidents investigated during the same corresponding sets of years."

$$\frac{\text{aver. attr. to pesticides 2007 to 2012}}{\text{aver. attr. to pesticides 2002 to 2007}} = 1\tfrac{2}{3} * \frac{\text{aver. incidents inv. 2007 to 2012}}{\text{aver. incidents inv. 2002 to 2007}}$$

Step 2: Next, I look at the graphs and find the correct numbers to substitute into my equation.

The missing points were said to be **"the same"** so I assign them each the variable *y*.

$$\frac{\left(\frac{2+5+9+14+y+y}{6}\right)}{\left(\frac{5+8+4+1+2+2}{6}\right)} = 1\tfrac{2}{3} * \frac{\left(\frac{24+39+42+27+18+30}{6}\right)}{\left(\frac{30+27+33+21+15+24}{6}\right)}$$

Step 3: I simplify and solve, first adding up the numbers in the parentheses, then performing the rest of the calculations.

$$\frac{\left(\frac{30+2y}{6}\right)}{\left(\frac{22}{6}\right)} = 1\tfrac{2}{3} * \frac{\left(\frac{180}{6}\right)}{\left(\frac{150}{6}\right)}$$

$$\left(\frac{30+2y}{6}\right) * \left(\frac{6}{22}\right) = \frac{5}{3} * \frac{30}{25}$$

$$\frac{6(30+2y)}{132} = \frac{5}{3} * \frac{6}{5}$$

$$\frac{180+12y}{132} = 2$$

$$\frac{180+12y}{132} = \frac{2}{1}$$

$$1\,(180+12y) = 2\,(132)$$

$$180 + 12y = 264$$

$$12y = 84$$

$$y = 7$$

Now I know the missing numbers. Each one is 7!

Step 4: I find the total number of bee colony collapses, for 2002–2012, by adding up all the numbers in that set.

$$\text{Total} = 5 + 8 + 4 + 1 + 2 + 2 + 5 + 9 + 14 + 7 + 7$$

$$\text{Total} = 64$$

Answer: The total number of bee colony collapses in County A, North Dakota from 2002–2012 was **64**.

THE HARDEST MATH PROBLEM

CHALLENGE 2 ANSWER KEY — GRADE 7

Although the problem has one correct numeric solution, there are multiple pathways students can take to arrive at the answer.

Step 1: I begin by calculating the cost of one bottle of each product.

Each bottle contains 64 oz. I will convert gallons (the bulk cost given) to ounces to find the per-bottle cost.

$1 \text{ gal} * \frac{4 \text{ qt}}{1 \text{ gal}} * \frac{2 \text{ pints}}{1 \text{ qt}} * \frac{2 \text{ cups}}{1 \text{ pint}} * \frac{8 \text{ oz}}{1 \text{ cup}} = 128 \text{ oz.}$ One gallon = 128 oz. Each bottle is ½ gallon

Product		Shipping Cost	Cost of 1 bottle
MintMix:	$\frac{\$34.29}{1 \text{ qt}} * \frac{4 \text{ qt}}{1 \text{ gal}} * \frac{1 \text{ gal}}{128 \text{ oz}} * \frac{64 \text{ oz}}{1 \text{ bottle}} = \frac{\$68.58}{1 \text{ bottle}}$	\$1.15	\$68.58 + \$1.15 = \$69.73
ZenEarthinol:	$\frac{\$88.50}{1 \text{ gal}} * \frac{1 \text{ gal}}{128 \text{ oz}} * \frac{64 \text{ oz}}{1 \text{ bottle}} = \frac{\$44.25}{1 \text{ bottle}}$	\$2.50	\$44.25 + \$2.50 = \$46.75
Mito-Down:	$\frac{\$130.60}{2.5 \text{ gal}} * \frac{1 \text{ gal}}{128 \text{ oz}} * \frac{64 \text{ oz}}{1 \text{ bottle}} = \frac{\$26.12}{1 \text{ bottle}}$	\$1.75	\$26.12 + \$1.75 = \$27.87
VarroAway:	$\frac{\$124.80}{1.5 \text{ gal}} * \frac{1 \text{ gal}}{128 \text{ oz}} * \frac{64 \text{ oz}}{1 \text{ bottle}} = \frac{\$41.60}{1 \text{ bottle}}$	(0.05)(\$41.60)= \$2.08	\$41.60 + \$2.08 = \$43.68
Garden+:	$\frac{\$75.16}{1 \text{ gal}} * \frac{1 \text{ gal}}{128 \text{ oz}} * \frac{64 \text{ oz}}{1 \text{ bottle}} = \frac{\$37.58}{1 \text{ bottle}}$	Free	\$37.58
NoPest:	$\frac{\$223}{2 \text{ gal}} * \frac{1 \text{ gal}}{128 \text{ oz}} * \frac{64 \text{ oz}}{1 \text{ bottle}} = \frac{\$55.75}{1 \text{ bottle}}$	(\$0.05)(64)= \$3.20	\$55.75 + \$3.20 = \$58.95

Step 2: Next, I find the combinations of 3 different bottles that have a total cost less than or equal to the budget amount of $120.

I consider the most expensive bottle, MintMix, at a price of $69.73. I subtract it from the club's budget:

$120.00 – $69.73 = $50.27

The sum of the two remaining bottles within this set must be less than or equal to $50.27 to stay within budget. If I subtract the lowest priced bottle ($50.27 – $27.87 = $22.40), I find there is not enough money left for any of the others to be the third bottle.

So, I eliminate this bottle, MintMix, as being a possibility in the final set of 3.

I use the same set of steps and reasoning to consider the next most expensive bottle, NoPest, which costs $58.95.

$120.00 – $58.95 = $61.05 $61.05 – $27.87 = $33.18

I cannot get a third bottle for less than or equal to $33.18, so I also eliminate NoPest as a possibility, too.

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THE HARDEST MATH PROBLEM
CHALLENGE 2 ANSWER KEY — GRADE 7

Although each problem does have a correct numeric solution, there are multiple pathways students can take to arrive at the answer.

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There are only 3 possibilities that will give a total less than or equal to the club budget of $120.

$46.75 + $27.87 + $43.68 = $118.30 (Zen / Mito / VarroA)

$46.75 + $27.87 + $37.58 = $112.20 (Zen / Mito / Garden+)

$27.87 + $43.68 + $37.58 = $109.13 (Mito / VarroA / Garden+)

Step 3: Now, regarding the two combinations with the same median increase in honey production, I must find the one with the lower cost per ounce.

The median of a data set is the middle value when the data is arranged in ascending order (from low to high).

I organize the percent increases in honey production:

Zen/Mito/VarroA	58%, 71%, 99%	median = 71%
Zen/Mito/Garden+	58%, 71%, 95%	median = 71%
Mito/VarroA/Garden+	71%, 95%, 99%	median = 95%

Since the first two have the same median, I have to find the one that has “the lowest average cost per ounce.”

To find the average cost per ounce, I will find each product’s cost per ounce, then average those 3 unit rates.

Zen/Mito/VarroA $$\frac{\left(\frac{\$46.75}{64}\right)+\left(\frac{\$27.87}{64}\right)+\left(\frac{\$43.68}{64}\right)}{3} = \frac{\$1.8484375}{3} = \$0.6161458\ldots \approx \$0.62$$

Zen/Mito/Garden+ $$\frac{\left(\frac{\$46.75}{64}\right)+\left(\frac{\$27.87}{64}\right)+\left(\frac{\$37.58}{64}\right)}{3} = \frac{\$1.753125}{3} = \$0.584375 \approx \$0.58$$

I find that the combination of Zen/Mito/Garden+ has the lower unit cost per ounce.
I could also have skipped calculating cost per ounce since there is only 1 bottle that is different in the combinations. One has VarroA and the other Garden+. Since both bottles contain 64oz and the Garden+ bottle costs less than the VarroA, I know the average cost per ounce will be less in the combination with Garden+.

Step 4: Looking back at my previous calculations in step 2, I see that the total cost of 1 bottle each of ZenEarthinol, Mito-Down, and Garden+ is $112.20.

Answer: The 7th graders spent a total of **$112.20** on bee-friendly pesticides.

THE HARDEST MATH PROBLEM
CHALLENGE 2 ANSWER KEY — GRADE 8

Although each problem does have a correct numeric solution, there are multiple pathways students can take to arrive at the answer.

Step 1: I need to find the **"total overall profit,"** so I start with the profit equation.

Profit = Sales – Expenses

Step 2: Next, I set up the equations for **sales** and **expenses**.

Let x = number of bottles   Let m = cost of making enough concentrate to fill one bottle

Each bottle is sold for $9.72, so: **Total Sales = $9.72x**

Total Expenses consist of variable expenses and fixed costs.

Variable expenses are those that *vary* according to the number of items made.

Fixed costs are the start-up, or one-time costs.

Total Expenses = (\$1.45x + \$0.05x + (m)x) + (\$65.88 + \$55 + \$26)

- *variable expenses*: \$1.45x (Empty bottle) + \$0.05x (label) + (m)x (cost (\$) of pesticide concentrate)
- *fixed costs*: \$65.88 (mixer) + \$55 (pans) + \$26 (kitchenware)

Total Expenses = \$1.50x + mx + \$146.88

Step 3: Now, I'll look for the break-even point in my data or graph. This is the point where there is no gain or loss. In other words, sales equal expenses.

On a graph, it's the point where the two rays or lines intersect.

Jade's sketch shows the rays intersecting at 34, so this is the break-even point.

Step 4: I substitute the value of x into my equation to solve for m, the cost of making enough concentrate to fill 1 bottle.

I substitute the break-even point, x = 34.

$$\text{Sales} = \text{Expenses}$$
$$\$9.72(34) = \$1.50(34) + m(34) + 146.88$$
$$\$330.48 = \$51 + 34m + 146.88$$
$$\$330.48 = 34m + \$197.88$$
$$\$132.60 = 34m$$
$$\$3.90 = m$$

The cost of making just the concentrate is \$3.90/bottle. I have to add in the empty bottle and label, too.

\$3.90 + \$1.45 + \$0.05 = \$5.40 cost to make 1 complete bottle

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THE HARDEST MATH PROBLEM
CHALLENGE 2 ANSWER KEY — GRADE 8

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Step 5: When profits are greater than \$500, they will donate 2 bottles for every 15 sold. So, I need to find out how many bottles it takes to get to a profit greater than \$500.

$$\textit{Profit} > \$500$$
$$\textit{Sales} - \textit{Expenses} > \$500$$
$$\$9.72x - [(\$5.40x + \$146.88)] > \$500$$
$$\$9.72x - \$5.40x - \$146.88 > \$500$$
$$\$4.32x - \$146.88 > \$500$$
$$\$4.32x > \$646.88$$
$$x > 149.\overline{740}$$

Starting with the 150th bottle, they need to donate 2 bottles for every 15 bottles sold.

Step 6: I use another equation to find when they reach a total donation of 26 bottles.

I translate the words into an equation: They donate **26** bottles, which **equals 2 times** the **number of times 15 occurs between** the milestone of **150** bottles sold and the future milestone of an unknown number of bottles sold, ***y***. While I'm setting up the equation, I also realize that subtracting 150 from *y* only provides the difference between those numbers (the number of bottles sold AFTER the 150th bottle). I need to account for the first bottle sold in the club's donation plan (the 150th bottle), so I add the +1 below.

$$\textit{donations} = 2\left(\frac{(y-150)+1}{15}\right)$$

$$26 = 2\left(\frac{y-149}{15}\right)$$
$$13 = \frac{y-149}{15}$$
$$195 = y - 149$$
$$344 = y$$

When they've sold the 344th bottle produced, they will have donated 26 bottles to the nursing homes.

$$\text{Profit} = \text{Sales} - \text{Expenses} - \text{Sales of 26 bottles}$$
$$\text{Profit} = \$9.72(344) - [\$5.40(344) + 146.88)] - \$9.72(26)$$
$$\text{Profit} = \$3{,}343.68 - [\$1{,}857.60 + 146.88] - \$252.72$$
$$\text{Profit} = \$3{,}343.68 - [\$2{,}004.48] - \$252.72$$
$$\text{Profit} = \$1{,}339.20 - \$252.72$$
$$\text{Profit} = \$1{,}086.48$$

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SCHOLASTIC

THE HARDEST MATH PROBLEM

CHALLENGE 2 ANSWER KEY — GRADE 8

Although each problem does have a correct numeric solution, there are multiple pathways students can take to arrive at the answer.

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Step 7: Now, I'm ready to find the profit after selling the 344th bottle. Special note: No money was received for 26 of those bottles. I have to subtract that money in my calculations.

Step 8: I must figure out the profit per ounce of solution (not concentrate).
For each 12 oz bottle of concentrate, you need to mix 7 parts water to 1 part concentrate to get the actual amount of solution it will make. I set up and solve a proportion.

$$\frac{\text{12 ounces con.}}{n \text{ ounces in total}} = \frac{1}{7+1} \qquad \frac{\text{12 ounces con.}}{n \text{ ounces in total}} = \frac{1}{8} \qquad n = 12(8) \qquad n = 96$$

Each 1 bottle of concentrate makes 96 ounces of pesticide solution.

$$\frac{\text{profit}}{\text{ounce}} = \frac{\$1{,}086.48}{\text{344 bottles}} * \frac{\text{1 bottle}}{\text{96 ounces of solution}} = \frac{\$0.032899709...}{\text{1 oz solution}}$$

Answer: Their overall profit, per ounce of pesticide solution, when 26 bottles have been donated to the nursing homes, is **about $0.03 (or 3 cents) per ounce**.